THE EVOLUTION OF MAGIC SQUARES IN CHINA *

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MAGIC SQUARES are diagrams composed of numbers or letters, specially arranged in a rectangular pattern to produce certain distinctive effects. Although the lettered squares were very common in the Islamic world, they do not appear in China, so we shall here consider only the numbered ones. These numerical magic squares were normally composed of all the numbers from one to the square of the base number—for example, in the magic square of three, this would include the numerals from 1 to 3^2, or 9—and these are all carefully placed so that every row and every column, and the two main diagonals, will each add up to the same number. This common sum is usually called the Constant.

Although these numerical magic squares are now usually relegated to that minor subdivision of Western Science known as "mathematical recreations," people of past centuries, all over the civilized world, took them very seriously. Their construction was regarded as a worthwhile study in mathematical techniques, or even as a creative artistic expression (in cultures with strong aniconic traditions); and the finished squares were considered as solemn religious symbols, or as charms and talismans with innate powers for good or evil. As magical diagrams they played an important part in the occult studies which formed a branch of mediaeval proto-science among the Chinese, the Hindus, and the Arabs, and ultimately among the Europeans of the Early Renaissance. And, in working with them, men made discoveries regarding the relationships of numbers which led to significant developments in Mathematics.[1]

In view of the position which they occupied in Old China, Joseph Needham, in the third volume of his history of science in China, has devoted a section to the Chinese development of magic squares.[2] In this, he attempted to show the place of magic squares in Chinese mathematics, as studies in "combinatorial analysis," and then tried to compare the Chinese situation with the development of magic squares elsewhere, in order to demonstrate that Chinese progress in this field has been both continuous and impressive. In doing so, however, he quite misrepresented the actual story; thus, it seems worthwhile to cover the ground again, to try to put things back into their proper perspective.

Authoritative writers on the history of mathematics still frequently assert that magic squares are of great antiquity, and that they have been known from very ancient times in China and India;[3] although responsible scholars have long since pointed out that there is no evidence at all for an early Indian claim,[4] and that the Chinese traditions of a magic square of great antiquity are spurious.[5] However, even if the magic square does not go back as far in China as tradition has represented it, apparently the Chinese actually did invent the first magic square. The earliest clear reference to the well-known magic square of three appears in the *Ta Tai Li-chi*, compiled in the first

* The substance of this article was presented as a paper before the Chinese session of the Annual Meeting of the American Oriental Society, at Yale University, April 1960.

[1] A short, but quite inaccurate summary of the history of magic squares is presented in D. E. Smith, *History of Mathematics*, II (New York 1925 [and 1953]), 591-597. The general development of magic squares, with exaggerated emphasis on the Arabic contributions, was sketched by W. Ahrens, "Studien über die magischen Quadrate der Araber," *Der Islam*, VII (1917), 186-250.

[2] Joseph E. Needham, *Science and Civilization in China*, III (Cambridge, 1959), 55-62.

[3] See, for example, the introductory remarks in the article "Magic Square," in the *Encyclopaedia Britannica*, 1957 edition, XIV, 625.

[4] The apparent emptiness of the Indian claim, which had so long been taken seriously, was first pointed out by Siegmund Günther, in his *Vermischte Untersuchungen zur Geschichte der mathematischen Wissenschaften* (Leipzig, 1876), pp. 188-189. This was also stressed by Ahrens, *op. cit.*, pp. 193, and 217-219.

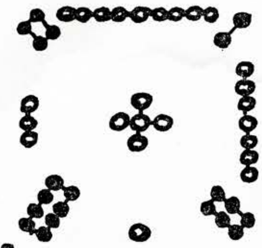
[5] For an example of the exaggerated claim for China (c. 2200 B.C.), see D. E. Smith, *History*, II, 591. Ahrens (pp. 191-192) went to the opposite extreme, trying to deny any Chinese knowledge of magic squares before mediaeval times, in order to try to prove the priority of the Arabs, who did so much to develop them.

1	4	7
2	5	8
3	6	9

1.

4	9	2
3	5	7
8	1	6

2.



3.

四	九	二
三	五	七
八	一	六

4.

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

5.

1	23	16	4	21
15	14	7	18	11
24	17	13	9	2
20	8	19	12	6
5	3	10	22	25

6.

13	22	18	27	11	20
31	4	36	9	29	2
12	21	14	23	16	25
30	3	5	32	34	7
17	26	10	19	15	24
8	35	28	1	6	33

7.

1	10	19	28	37	46	55	64	73
2	11	20	29	38	47	56	65	74
3	12	21	30	39	48	57	66	75
4	13	22	31	40	49	58	67	76
5	14	23	32	41	50	59	68	77
6	15	24	33	42	51	60	69	78
7	16	25	34	43	52	61	70	79
8	17	26	35	44	53	62	71	80
9	18	27	36	45	54	63	72	81

8.

31	76	13	36	81	18	29	74	11
22	40	58	27	45	63	20	38	56
67	4	49	72	9	54	65	2	47
30	75	12	32	77	14	34	79	16
21	39	57	23	41	59	25	43	61
66	3	48	68	5	50	70	7	52
35	80	17	28	73	10	33	78	15
26	44	62	19	37	55	24	42	60
71	8	53	64	1	46	69	6	51

9.

1. The natural square of 3; "Theon's square."
2. The magic square of 3 (the Nine Halls), usual form.
3. Pseudo-archaic rendering of the *Lo Shu* Diagram.
4. Magic square of 3 (the Nine Halls), in ordinary Chinese rendering.
5. The natural square of 5, with center emphasized.
6. Yang Hui's first magic square of 5, with *Lo Shu* core. (after Li Yen).
7. Yang Hui's first magic square of 6, in *Lo Shu* pattern. (after Li Yen).
8. The natural square of 9, emphasizing the horizontal rows.
9. Yang Hui's magic square of 9, composed of nine small *Lo Shu* squares. (after Li Yen).

century A. D. from older sources. This lists the numbers in the order of their appearance in the square, while describing the nine rooms or halls of the cosmic temple building known as the Ming T'ang.[6] Because of this, later Chinese scholars often spoke of the mathematical lore surrounding this square of three as the "Nine Halls calculations" (*chiu-kung suan*[a]).

Various modern Western authors, including Needham, have asserted that the earliest magic square known elsewhere was given by Theon of Smyrna, a Neo-Pythagorean, about 130 A. D.[7] However, if they had looked up this so-called "magic square" of Theon's, they would have seen that it was not a magic square in any sense of the term. Theon merely presented the "natural square" of three; that is, he just listed the numbers from 1 to 9 in three columns, in their regular order.[8]

In view of the great emphasis on theories of numbers and numerical symbolism among the Pythagoreans, beginning with Pythagoras himself, it might be expected that some of them, or at least their Neo-Pythagorean successors, might have invented magic squares; but we have no definite evidence that any of them ever did. The known facts at present would seem to show that the earliest magic squares outside of China did not appear until about the ninth century A. D. At that time the Arab world began to take great interest in the square of three, possibly having received it directly from China in the course of the extensive Arab-Chinese trade relations during the T'ang Dynasty.[9] Thus, it appears that the magic square in China did indeed have a priority of at least a thousand years; even though it did not go back as far as many previous writers have implied.

An extensive folklore and corpus of magical beliefs gradually grew up around the square of three in China, and it was already being employed in fortune-telling in the second century A. D., by the Later Han mathematician Chang Hêng,[b] who used the "Nine Halls" in his system of divination.[10] This is not the place to discuss all the deep meanings and fancied powers ascribed to it, as our interest here is—like Needham's—purely in the development of magic squares as solutions to mathematical problems of balance and placement. We can only pause long enough to remark that the mediaeval Chinese associated the magic square of three with the diagram that was supposed to have appeared on the shell of a sacred turtle, which, according to legend, appeared to the mythical King Yü from the waters of the Lo River at the time of the taming of the floods. As such, in mediaeval times and later, the Chinese called it the *Lo Shu*[c], or "Document of the Lo River," and, in token of its supposed antiquity, they generally represented it by a pseudo-archaic arrangement of black and white knots or beads on short lengths of cord—Only the Tibetans of West China and Tibet itself, and the Mongolian lamas, have preserved graphic representations of the magic square of three figured on the body of a turtle, and they have pictured this on divination charts, and occult paintings down to the present time.[11]

Having had such a long head-start, the Chinese might have been expected to go on to develop magic squares of higher numbers at a comparatively early date—and probably they did. However, Needham describes the further development of the magic square in China—which he represents as a continuous progress of advancing skill—as beginning with a scholar named Yang Hui[d], who lived in the thirteenth century, toward the end of the Sung Dynasty. Needham seems to regard Yang Hui as having been something of a mathe-

[6] *Ta Tai li-chi*[k], no. 67 (*Ming T'ang*).

[7] Needham, III, 61. Probably he got this impression from George Sarton, *Introduction to the History of Science*, I (Baltimore, 1927), where Theon is credited with giving the earliest suggestion of a magic square, excepting the Chinese tradition.

[8] "Theon's square," in the setting of the original Greek text, was reproduced in J. Dupuis, *Théon de Smyrne* (Paris, 1892), p. 166, with the translation facing it on p. 167. Of course, in the original, Greek letters were used in place of numbers. The first book to suggest that Theon's square might have magical properties seems to have been G. Loria, *Le Scienze esatte nell'Antica Grecia* (Milan, 1914), p. 795; but the fact that it was not a magic square in any sense was conclusively demonstrated by Ahrens, pp. 193-194.

[9] The Arabic scholar Tābit ibn Korra (ibn Merwān el-Harrānī, A. D. 836-901) seems to have been the earliest writer outside of China to discuss magic squares. See Heinrich Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke* (Abhandlungen zur Geschichte der Mathematischen Wissenschaften, X), p. 34, no. 66.

[10] See his biography in the *Hou Han shu*, ch. 89, 10b.

[11] See A. K. Gordon, *Tibetan Religious Art* (New York, 1952), p. 27 for one of these paintings. The description of it on p. 29 says the magic square, etc., are figured on a frog; but the serrated edges of the turtle shell, and the creature's tail below, easily identify it. Incidentally, the picture is printed backwards.

matical genius, worthy of comparison with his Byzantine contemporary, Manuel Moschopoulos, who is usually credited with having been the first person to discuss magic squares on a purely mathematical basis, as well as being the first to introduce magic squares to the Byzantine world, and ultimately to the rest of Europe.[12]

Actually there is no real comparison between these two men, except that both were apparently only transmitting the mathematical works of other men. Moschopoulos was primarily a philologist and translator, and we now know he was passing on the work of some Persian scholar (or scholars) as yet unidentified; [13] while Yang Hui himself admitted in the preface to his book which contained the squares (the *Hsü-ku chai-chi suan-fa*[e], of 1275 A. D.) that he was merely handing down the works of men of old, and he made no claim to being any sort of an innovator himself.[14] He did not even describe the methods of constructing them, as Moschopoulos did, and perhaps he could not have done so. In fact, if one carefully examines the magic squares which Yang presents, it is obvious that they belong to several different systems, not all of which were truly Chinese, some of them having already appeared in the Islamic world nearly three centuries before; and, in comparison with the methods illustrated by Moschopoulos, their construction seems relatively primitive. Yang referred to them as *tsung-hêng t'u*[f], literally: "vertical-horizontal diagrams," and Chinese scholars of later generations continued to use this term.

The elaboration of magic squares, using higher numbers, may possibly have begun in the Han Dynasty; but it was undoubtedly developed during the Six Dynasties (fourth to sixth centuries A. D.), when learning was being quietly maintained in the monasteries, secluded from the turmoil of "China's Dark Ages," and men took considerable interest in divination in an attempt to find some hope in a time of general despair. Several works on the Nine Halls and the *Lo Shu* are said to have been written during this period, although they were later destroyed, and we only know them from brief quotations in later works.[15] No doubt further developments were made during the T'ang and early Sung dynasties, when foreign trade brought new ideas, including new mathematical concepts from Western Asia and India, and probably even actual magic squares.

In the absence of surviving records, we are still unable to trace, step by step, the course of that early evolution of magic squares in China. We can only view its results as represented in the group of magic squares passed down to us by Yang Hui's book, which still remains the earliest corpus of advanced magic squares yet known from China. To study these magic squares properly, we must go back to the examples themselves; which Needham apparently did not do. Luckily, they have recently been resurrected for modern students by a Chinese historian of mathematics, Li Yen, who was fortunate enough to possess an edition of Yang Hui's rare book in his own private library.[16]

Needham repeatedly quotes Li Yen,[17] but his various misstatements about Yang Hui, and those who followed him, show all too clearly that he had neither read Li Yen's treatise carefully, nor looked at the magic squares themselves, which are after

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[12] For the life of Moschopoulos, see G. Sarton, *Introduction to the History of Science*, part 3.1 (Baltimore, 1947), 679-680. For Moschopoulos' methods of construction, presented in a letter written to his friend Nicholas Rhabdas, c. 1300, see Paul Tannery, *Mémoires Scientifiques*, IV (Paris and Toulouse, 1920), 27-60, which contains both the original Greek text, and a French translation, with actual examples.

[13] The two principal methods of constructing magic squares described by Moschopoulos, and usually accredited to him, had already been used in Persia. Examples are shown in an anonymous Persian manuscript of A. D. 1212, in Princeton University (Garrett Collection, no. 1057).

[14] Li Yen, *Chung suan-shih lun-ts'ung*,[l] III (Shanghai, 1934), p. 61, quotes Yang Yen's original preface, before presenting reproductions of his magic squares, in Western style, with "Arabic" numerals.

[15] These appeared in a category of works known as *wei shu*, later strictly proscribed.

[16] See reference in note 14, above.

[17] Needham calls him Li Nien, throughout, and gives some rather confused references to his work. For instance, in his Bibliography (III, 731), under "Li Nien (4)," he cites this book, the *Chung suan-shih lun-tsung*, commenting that a fourth volume, in two parts, was published in 1947. Actually this "fourth volume" was another book under the same title, containing substantially the same material as the previous three-volume work, in different order. For example, the treatise on magic squares, which appeared in volume 3 of the first book, reappears with only very slight additions in volume 1 of this second book (pp. 175 ff.). And the entry under "Li Nien (21)" (p. 732), bearing the same name as the two preceding ones, is not a "second series," as listed, but is apparently the same material being presented for a third time.

all the basic evidence for a discussion of this subject. For example, in trying to give his readers the impression that Yang Hui had made an important contribution to the development of magic squares in treating them purely as mathematical problems—without superstitious considerations—Needham makes the statement that Yang had given some simple rules for their construction.[18] Had he really read Li Yen carefully, he would have seen that the only explanations of construction (with the exception of brief directions for making the simple squares of three and four) were actually Li Yen's own attempts to work out the methods that had been used in making the squares presented by Yang Hui.[19] Li makes it clear that Yang Hui himself had given no explanations of how to make the squares he had presented—beyond those of three and four. Indeed, as we have remarked, it seems quite likely that Yang Hui did not know how to do all of them himself, but was merely passing on the finished diagrams from older books.

We shall not discuss here in any detail the squares presented by Yang Hui, as I intend to do that in another article; but we can take a quick review of them to see their principal features, and to note some facts that have special bearing on the Chinese development of magic squares.

Yang Hui began by giving the familiar square of three—the *Lo Shu* or *chiu kung* square—in its most usual form. Then he presented two magic squares of four,[20] both of which were variations on one already known long before in the Arabic world, having been published in the encyclopaedia of the Brethren of Purity (the *Rasā'il* of the Ikhwān al-Ṣafā) about 990 A.D.[21] Thus these four-squares were most probably foreign borrowings, along with Yang Hui's two magic squares of eight (the second of which was badly misunderstood). Since these were apparently outside the Old Chinese heritage, I shall not attempt to describe them further, here.

In the two magic squares of five presented by Yang Hui,[22] the first was apparently constructed by taking the "natural square" of five, that is to say, the numbers from 1 to 25 written in five parallel rows (or columns), then rearranging the nine numbers at the center following the order of the *Lo Shu* (inverted); after which the other sixteen numbers in the outer border were also re-arranged, so that the complementary pairs stood opposite each other, in order to complete the common sums. This use of the *Lo Shu* pattern to form the core of what is technically known as a "bordered magic square" is a very characteristic Chinese solution, and it was probably already very old in China. However, the first published example also goes back to the Arabs,[23] and the method was later used by them in far more sophisticated and efficient ways, the possibilities of which the Chinese themselves apparently never learned.[24]

Yang's second five-square is more laboriously constructed, by a careful positioning of all the complementary pairs around the middle number of the original sequence, which occupies the center of the square. With this method of placing, the two numbers of each opposing pair can be connected by a line through the center number, and each of those pairs adds up to twice the sum of the center number, making what is technically called an "associated magic square." This not only represents a style of construction already known and used by the Brethren of Purity, nearly three hundred years before, but it also has a fur-

[18] Needham, III, 59.

[19] Li Yen, 3, pp. 71-77, devotes a section to his own reconstructions of Yang Hui's squares. Even these are rather superficial, as—especially in those of the higher numbers—he failed to observe some important phenomena within the squares themselves, and misinterpreted the construction methods.

[20] *Ibid.*, pp. 62-63.

[21] Reproduced in F. Dieterici, *Die Propadeutik der Araber* (Berlin, 1865), p. 43, fig. 2. Ahrens, who knew only the first four of the seven magic squares presented in the *Rasā'il*, tried very hard to prove that they were spurious, or later additions (Ahrens, pp. 205-213); however, a careful study of the complete set, and their methods of construction, shows clearly that they were indeed primitive, and quite consistent with the stage of Arab knowledge in this field at the end of the 10th century, and therefore they are very likely an integral part of the original work.

[22] Li Yen, p. 64.

[23] The first appearance of a bordered square was the double-bordered square of seven in the *Rasā'il* of the Ikhwān al-Ṣafā, shown in the Cairo edition of 1928, I, 70.

[24] Li Yen, vol. 3, p. 102, reports finding an anonymous manuscript of the Early Ch'ing period (17th century) in the Palace Library in Peking, describing a general solution for magic squares by a bordered square method already well known in Europe, probably composed by one of the Roman Catholic missionaries then residing in Peking. As the method was apparently never adopted in China, it was probably discarded as a mere foreign curiosity.

ther characteristic common in Arabic magic squares of the twelfth and thirteenth centuries: namely, the augmenting of all the numbers;[25] for each number in this square had been increased by 8, beforehand, so the sequence ran from 9 to 33, instead of going from 1 to 25, as it usually does. If the two features of association and augmentation were the only qualities that distinguished this square, it might also be dismissed as a borrowing from the Islamic world; but typically Chinese reasons for both the construction and the addition of that particular number, 8, can be found in the Old Chinese philosophical and numerological ideas concerning magic squares (as I hope to explain elsewhere), so there need be no question that it was a native Chinese magic square. Quite possibly it was even older than the Arabic ones of the same general type, because there is evidence that Yang Hui no longer fully understood it. It was apparently totally incomprehensible to his successors, who dropped it from the Chinese repertory of magic squares, doubtlessly because all they saw in it was a comparatively difficult means of construction.

The magic square of six has always offered problems to mathematicians elsewhere (except to the Arabs, who early found some clever ways to cope with it), and it is still called "the most difficult magic square of all," and "the most complicated of all magic squares to construct." (This is because 6 is an "oddly-even" number, rather than being "doubly-even" like 4 and multiples of 4.) The Old Chinese, faced with this problem which caused so much trouble to scholars elsewhere, solved it rather ingeniously by grouping the numbers of the sequence from 1 to 36 in such a way that they could be fitted into a *Lo Shu* pattern, taking advantage of a system with which they felt entirely at home. Two variations of this were presented by Yang Hui.[26] Apparently it did not occur to anyone else, until modern times, that an even-numbered magic square could be constructed on the basic pattern of an odd-numbered one, and the modern solution is quite different; so this was a uniquely Chinese development.

In the case of the two magic squares presented by Yang Hui,[27] the first solution once more drew upon the *Lo Shu* pattern, as the central core was made from nine numbers taken at regular intervals from the natural square of seven, with the other numbers arranged around this in a double frame. It is important to note, however, that this is not what is technically known as a "double-bordered magic square," of the type in which each border or frame can be stripped away successively, always leaving a perfect magic square; as the second or inner border is defective. Strangely enough, the Chinese never seem to have had a true double-bordered square of seven, although the Brethren of Purity had published one long before the time of Yang Hui,[28] and the Arabs and Indians used them, in numerous variations, from the twelfth century on.[29]

Yang's second magic square of seven was another associated one, constructed on a *Lo Shu* base, like the second square of five, but without the augmentation of its numbers. This, too, was apparently not appreciated by Yang Hui's sucessors, so they dropped it when they discarded the second five square, possibly because they considered it foreign.

Yang Hui's magic square of nine shows the greatest development of the *Lo Shu* principle as evolved by the Old Chinese mathematicians.[30] Apparently, the man who invented it first wrote down the sequence of numbers from 1 to 81 in nine parallel columns, after which he took each horizontal line of nine numbers, in turn, and set them out in the order of their sequence, to make from each row a small square on the *Lo Shu* pattern. Finally, he took these nine subsidiary squares in order and arranged them to construct a much larger square, again on the *Lo Shu* pattern. The result was a "composite magic square," consisting of nine smaller magic squares, each capable of functioning separately.

Possibly the inventor of this square of nine was Chang Hêng, because later Chinese scholars who commented on his biography, before the early lore was entirely lost, described his method of divination with the Nine Halls as using the numbers

[25] This augmenting of numbers is frequently found in the magic squares of al-Būnī (Ahmed ibn 'Alī ibn Yūsuf al-Būnī, d. 1225), which have appeared in numerous modern editions of his works.

[26] Li Yen, vol. 3, p. 65.

[27] *Ibid.*, pp. 65-66.

[28] The bordered square referred to in note 23.

[29] For an elaborate use of these by al-Būnī, see Carra de Vaux, "Une solution arabe du problème des carrés magiques," *Revue d'Histoire des Sciences*, I (1948), 206-212.

[30] Li Yen, vol. 3, p. 67.

from 1 to 9, or from 1 to 81.[31] Actually, in spite of its apparent complexity, and many other rather amazing features not discussed here, this is one of the easiest magic squares to make, as it is only a logical extension of the simple square of three. As soon as people began to experiment with the latter, they might quite naturally have happened upon this one.

Yang Hui's square of ten[32] starts out like a characteristically Indian type used for "oddly-even" numbers, which was probably already old when Nārāyaṇa Paṇḍita presented several good examples of such squares in 1356; but here it was not carried to its full conclusion, so this is not a true magic square. It is only "semi-magic," as its diagonals will not add up right. As such, it seems an excellent example of a foreign borrowing, misunderstood.

Apparently the mediaeval Chinese took little interest in even-number magic squares and were very weak in handling them. For we have seen that they only had six workable magic squares of even numbers, two of which were solved by an adaptation of the method they used so frequently for odd numbers, the *Lo Shu* principle, and four of which seem to have been foreign borrowings. This should not greatly surprise us. In the first place, for various cultural and philosophic reasons, the mediaeval Chinese were only interested in the *Lo Shu* and the variant applications of it; and secondly, there had been plenty of foreign influence close at hand for several centuries. Yang Hui himself lived in the region of Hangchow,[33] which was then the political and economic capital of the Sung Dynasty, and a noted seaport and center for international trade. As a very cosmopolitan city, it had long had a remarkably mixed and varied population, which included Arabs, Persians, and Indians who had come to China as merchants.

Contrast this with the position of Ch'êng Ta-wei[g], who is mentioned by Needham as the next important producer of magic squares. Ch'êng came from the interior province of Anhwei, in the Ming Dynasty,[34] when reaction against the recent alien rule of the Mongols had produced strong anti-foreign feelings, and every effort was being made to "purify" the Old Chinese tradition by returning to the ways of the T'ang and the Sung periods. Ch'êng Ta-wei was no producer, or innovator, as Needham and other modern writers have stated. He, too, was merely a collator and transmitter, as were so many of his contemporaries, and primarily he was merely handing on the magic squares of Yang Hui. By his time, these were probably no longer understood, nor fully appreciated, so he simplified the original selection of Yang Hui's, retaining only one of each pair in the cases where Yang Hui had presented two squares for a single number, and his choice was not always the wise one.

A too-hasty glance at Li Yen's presentation of Ch'êng Ta-wei's magic squares (which, incidentally, were also reproduced in the K'ang-hsi Encyclopaedia[35]) might give the impression that Ch'êng had created some new squares, but the only innovations were slight variations on Yang's first square of five and his second square of six.[36] The former does not work at all, as its top and bottom rows will not add up properly; while the latter is merely Yang Hui's second six-square split vertically down the middle with the two halves exchanged and rejoined; but, in altering it, Ch'êng destroyed the *Lo Shu* form and thus relinquished some further characteristics that Yang's six-square had originally possessed. Lastly, in some of his explanations for the others, he showed that he did not have the vaguest idea of their actual method of construction.

In short, Ch'êng Ta-wei, far from being an innovator in the field of magic squares, did not rightly understand the old ones he had borrowed. Therefore, he certainly does not deserve the title frequently bestowed on him by modern writers, as an important Chinese inventor of magic squares. Ch'êng's selection represents only a link in the line of transmission from Yang Hui to later mathematicians of Ch'ing dynasty China and Tokugawa Japan.[37] Even as a transmitter he was rather ineffective, as he failed to pass on several

[31] *Hou Han shu*, ch. 89, 10b.

[32] Li Yen, vol. 3, p. 68.

[33] His home was at Ch'ien-t'ang, in modern Chekiang province.

[34] For Ch'êng Ta-wei's biography, see A. W. Hummel, ed., *Eminent Chinese of the Ch'ing Period*, I (Washington, D. C., 1943), 117.

[35] The *T'u-shu chi-ch'êng* reproduces the magic squares from Ch'êng Ta-wei's book, *Suan-fa t'ung-tsung*[m], in its mathematical section, *li-fa tien*, ch. 125, *suan-fa pu*[n], 17.

[36] Reproduced in Li Yen, vol. 3, p. 78.

[37] For Ch'êng Ta-wei's influence in Japan, as regards magic squares, see Yoshio Mikami, *The Development of Mathematics in China and Japan* (Abhandlungen zur

of Yang Hui's more interesting squares, and spoiled two of the ones that he did pass on, thus impoverishing the tradition for later generations.

Needham goes on to say that a few more magic squares were added by Fang Chung-t'ung[h] in 1661, and many more by his contemporary Chang Ch'ao[i], another Ch'ing Dynasty scholar.[38] However, on inspecting their actual work, as reproduced by Li Yen, it is clear that Fang merely copied Ch'êng Ta-wei's squares, except for a "new" five-square, which is just a slight variation—with no improvement—on Yang Hui's old one,[39] probably arrived at while trying to make Ch'eng's faulty one function properly again. Meanwhile, Chang Ch'ao's only innovation was a new magic square of ten,[40] which was very clumsily and unsystematically constructed, inferior in every respect to the Hindu, Near Eastern, and European ones of the same period.

Lastly, Needham cites a Ch'ing scholar of the latter part of the nineteenth century, Pao Ch'i-shou[j], claiming that he had published some three-dimensional magic squares.[41] This term "three-dimensional magic squares" implies "magic cubes," which by that time had already been discovered and published in the Western world. However, examination of the actual examples shows them to be only numbered boxes in outline, related to the outlined orbs that were developed in China from magic circles; they have nothing to do with magic squares and are far more primitive.[42] Incidentally, one reason for the failure of the Chinese to develop magic squares any further than they did was probably their later, great interest in the magic circles, which they also seem to have invented.[43] The latter are possibly more "artistic," but they are less capable of variation and elaboration of techniques, and are not nearly so significant from the point of view of mathematics.

In the meantime, before 1200, and hence before the birth of Yang Hui himself, the Arabs and Hindus had begun to invent more comprehensive methods that could be used to solve any odd-number square, and other methods for successive even-numbered ones;[44] and by the early sixteenth century, before the time of Ch'êng Ta-wei, even the Europeans—who were very late in starting—had begun to discover faster methods.[45] Thus, in other lands it was no longer necessary to work out the magic square of each number as a separate, individual problem with its own distinctive solution, as the Chinese continued to do, right down into modern times.

To sum up, we have seen that the Chinese probably did indeed invent the magic square of three, centuries before anyone else; but they were apparently so hypnotized by this early solution, and by all the cosmic and magical properties ascribed to it, that they continued to try to adapt the *Lo Shu* principal to the solving of higher squares as well. From an evolutionary point of view, this was a blind alley; and it prevented them from going on to find quicker and more efficient methods. Secondly, although the early development of magic squares in China was indeed impressive, judging from the results that have come down to us, this must have reached its height some time before 1275, when Yang Hui published his examples of the early work. During the course of this development, some foreign methods seem to have been adopted, only to be dropped again during the great reaction in the Ming; and later, more significant, foreign discoveries were never adopted at all. In short, far from showing a continuous advance since 1275—as described by Needham—the Chinese apparently made no real progress since some time before that date; and the whole tradition gradually weakened as it declined. Meanwhile other people continued to make conspicuous advances, findings new and easier solutions to the old problems.

Thus, we find that the magic squares—like so many other scientific or quasi-scientific things in

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Geschichte der mathematischen Wissenschaften, XXX, Leipzig and New York, 1913), pp. 11-112, and 290.

[38] Needham, III, 60.

[39] Li Yen, vol. 3, p. 80.

[40] *Ibid.*, p. 91.

[41] Needham, III, 60.

[42] An example is given by Needham, *ibid.*, fig. 62; see also Li Yen, vol. 3, pp. 94-95, where several examples are shown.

[43] Needham published one example of a Chinese magic circle, *ibid.*, fig. 60, from among the earlier Chinese diagrams passed on by Yang Hui and reproduced by Li Yen (vol. 3, p. 70). Li Yen's examples show that even these deteriorated in later times, as they lack the central number; c. f. those of Chang Ch'ao, in *ibid.*, pp. 86, 87, 89.

[44] See reference in note 29 for a comprehensive method of c. 1200 A. D., demonstrated by al-Būnī, although probably invented by someone else.

[45] As illustrated in the 16th century writings of Michael Stifel and Adam Riese, as well as in the finished squares reproduced by Agrippa of Nettesheim.

Old China—followed an all-too-common course, which goes far toward helping to answer the familiar question: why did science and mathematics never achieve their full development in mediaeval China? The Chinese certainly had plenty of inventive genius, as illustrated by their numerous significant discoveries; but they were too often satisfied by preliminary results based on an early-discovered method, without trying to improve upon this.[46] And also, while there were times when the Chinese welcomed loans from other cultures, these were too often followed by long periods of excessive chauvinism and intolerance toward other peoples, when the foreign borrowings were discarded, regardless of their practical value, just because they were foreign. Then, without the cross-fertilization of new ideas, and lacking the initiative to experiment further, their initial progress gradually slowed to a halt.

[46] Old Chinese priorities in many fields of invention were neatly demonstrated, in a table of comparative dating of scientific discoveries, in Needham, I (1954), 246.

CHINESE CHARACTERS

a. 九宮算
b. 張衡
c. 洛書
d. 楊輝
e. 續古摘奇算法
f. 縱橫圖
g. 程大位
h. 方中通
i. 張潮
j. 保其壽
k. 大戴禮記, 明堂
l. 李儼, 中算史論叢
m. 算法統宗
n. 曆法典, 算法部

A SHORT HISTORY OF JAPANESE NAGAUTA MUSIC

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JAPANESE MUSIC can be traced back historically some fifteen hundred years and is noted in legends of presumably greater antiquity. Over this long period one can see the gradual development of many genres of musical expression. Until approximately the thirteenth century Buddhist singing and court orchestral music were predominant. Between the thirteenth and the sixteenth centuries these musics were supplanted by lute narrations, the aristocratic Nō drama music, and the accompaniments for a host of folk theatricals. It was during the seventeenth and eighteenth centuries that the remaining "great" traditions of Japanese music were developed. These were zither (koto) music, the diverse forms for the three-stringed lute (shamisen), and the music for the Kabuki theatre. The shamisen music form known as *nagauta*, literally "long song,"[1] stands at the center of this last purely oriental, i. e., pre-Western-influenced, period of Japanese history. It is with the background and growth of this important form that the following article deals.

Since the history of *nagauta* is intimately connected with the general growth of shamisen music, it is necessary first to consider the early forms of shamisen music in order to place *nagauta* in its proper historical matrix. When the shamisen first came to Japan from the Ryūkyū Islands (circa 1560) it seems to have been used as a substitute for the larger *biwa* lute used by the storytellers in the Ōsaka-Kyōto district.[2] Traditionally it is said that around 1610 Sawazumi Kengyō and/or Ishimura Kengyō, both *biwa* musicians, began to play *kumiuta* music on the shamisen.[3] In view of the fact that *kumiuta* consists of a suite of short lyric pieces and is not a narrative form and the

[1] The term *nagauta* or *chōka* is used in the ancient collection, the *Manyōshū* (circa A. D. 760) to indicate poems of greater than usual length. This term, however, has no known historical connection with the music under discussion in this article.

[2] Tanabe Hisao, *Nihon no ongaku* (Japanese Music) (Tōkyō, 1954), p. 288.

[3] Iba Takashi, *Nihon ongaku gairon* (An Outline of Japanese Music) (Tōkyō, 1928), p. 806.